Name:

SA402 · Dynamic and Stochastic Models | Fall 2023 · Uhan

Quiz 7 – 10/25/2023

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue calculator. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
2	1	
Total		/ 40

Problem 1. Vehicle arrive at the Simplexville Bridge toll plaza between 7 a.m. and 7 p.m. according to a nonstationary Poisson process with integrated rate function

$$\Lambda(\tau) = \begin{cases} 20\tau & \text{if } 0 \leq \tau < 2 \\ 5\tau + 30 & \text{if } 2 \leq \tau < 10 \\ 25\tau - 170 & \text{if } 10 \leq \tau \leq 12 \end{cases}$$

where τ is in hours, $\tau = 0$ corresponds to 7 a.m., and $\tau = 12$ corresponds to 7 p.m.

a. In words, briefly describe the meaning of $\Lambda(4) = 50$ in the context of this problem.

b. What is the probability that the 45th vehicle arrives at the toll plaza at or before 11 a.m.?

a. If exactly 100 vehicles have arrived by 4 p.m., what is the probability that 135 or fewer vehicles will arrive by 6 p.m.?

Problem 2. Vehicles arrive at the nearby Turing Tunnel toll plaza between 7 a.m and 7 p.m according to a nonstationary Poisson process with arrival rate function

$$\lambda(\tau) = \begin{cases} 9 & \text{if } 0 \le \tau < 3 \\ 3 & \text{if } 3 \le \tau < 9 \\ 7 & \text{if } 9 \le \tau \le 12 \end{cases}$$

What is the integrated rate function for this nonstationary Poisson process?

	$X \sim \text{Poisson}(\mu)$	$X \sim \text{Exponential}(\lambda)$	$X \sim \text{Erlang}(n, \lambda)$
pmf / pdf	$p_X(a) = \begin{cases} \dfrac{e^{-\mu}\mu^a}{a!} & \text{if } a = 0, 1, 2, \ldots \\ 0 & \text{o/w} \end{cases}$	$f_X(a) = \begin{cases} \lambda e^{-\lambda a} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$	$f_X(a) = \begin{cases} \dfrac{\lambda(\lambda a)^{n-1}e^{-\lambda a}}{(n-1)!} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$
cdf	$F_X(a) = \sum_{k=0}^{\lfloor a \rfloor} \dfrac{e^{-\mu}\mu^k}{k!}$	$F_X(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$	$F_X(a) = \begin{cases} 1 - \sum_{k=0}^{n-1} \dfrac{e^{-\lambda a}(\lambda a)^k}{k!} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$
expected value	$E[X] = \mu$	$E[X] = \dfrac{1}{\lambda}$	$E[X] = \dfrac{n}{\lambda}$
variance	$\text{Var}(x) = \mu$	$\text{Var}(X) = \dfrac{1}{\lambda^2}$	$\text{Var}(X) = \dfrac{n}{\lambda^2}$